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Rules for Exponents and Logarithms

Exponents: a and b are real, positive, and not equal to one or zero. x and y are real.

PRODUCT LAW: $a^x a^y = a^{x+y}$

QUOTIENT LAW: $a^x / a^y = a^{x-y}$

POWER LAW: $(a^x)^y = a^{xy}$

DEFINITIONS: $a^0 = 1$; $a^1 = a$;
$a^{-x} = 1/a^x$; $(a/b)^{-1} = b/a$

APPLICATIONS: $(ab)^x = a^x b^x$
$(a^x / b^y) = a^x b^{-y}$; $(a^x / b^y)^z = a^{xz} b^{-yz}$

PROPERTIES: $a^x = a^y \Leftrightarrow x = y$; $a^x = b^x \Leftrightarrow a = b$

ROOTS $a^{1/n}$ = nth root of a; i.e., $(a^{1/n})^n = a$; $a^{1/n} = \sqrt[n]{a}$

$$\left(\frac{a^x}{b^y}\right)^p = \frac{a^{xp}}{b^{yp}}$$

$$\left(\frac{a}{b}\right)^{-p} = \frac{b^p}{a^p}$$

$$\left(\frac{a^x}{b^y}\right)^{-p} = \frac{a^{-xp}}{b^{-yp}} = \frac{b^{yp}}{a^{xp}}$$

Logarithms: b is real, positive, and not equal to one or zero. M and N are real and greater than zero. b is called the base of the logarithm.
Definition: $x = b^y \Leftrightarrow y = \log_b x$

$\log_b 1 = 0$

$\log_b b = 1$

$\log_b b^x = x$

$b^{\log_b x} = x$

COMMON MISTAKES:

$\log_b M \bullet \log_b N \neq \log_b M + \log_b N$

$\log_b M / \log_b N \neq \log_b M - \log_b N$

$(\log_b M)^p \neq p \log_b M$

PRODUCT RULE: $\log_b (MN) = \log_b M + \log_b N$

QUOTIENT RULE: $\log_b (M/N) = \log_b M - \log_b N$

EXPONENT RULE: $\log_b M^p = p \log_b M$

$\log_b M = \log_b N \Leftrightarrow M = N$;

NOTATION : $\log x = \log_{10} x$; $\ln x = \log_e x$,

Applications: Doubling/ Halving:

$A(t) = A_o 2^{t/t_{double}}$, where t_{double} = time for doubling such as the time to double an investment

$A(t) = A_o (½)^{t/t_{half}} = A_o 2^{-t/t_{half}}$ where t_{half} = time for halving such as half-life of a decaying isotope

Compound Interest Rate:

$A(t) = P_o (1 + (r/m))^{tm}$; r = interest rate in decimals, m = number of times per period (usually one year) interest is given, P_o = principal

Continuous Compounding (limit $m \to \infty$): $A(t) = P_o e^{rt}$